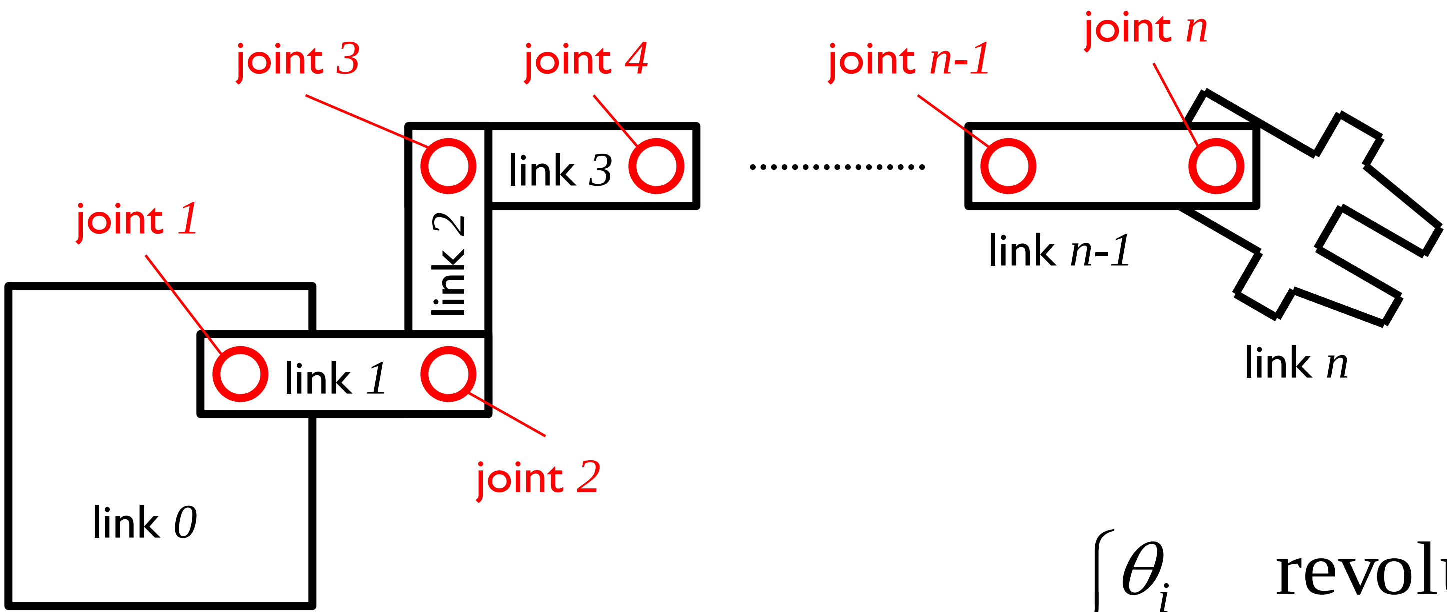
Day 05

Forward Kinematics

Links and Joints

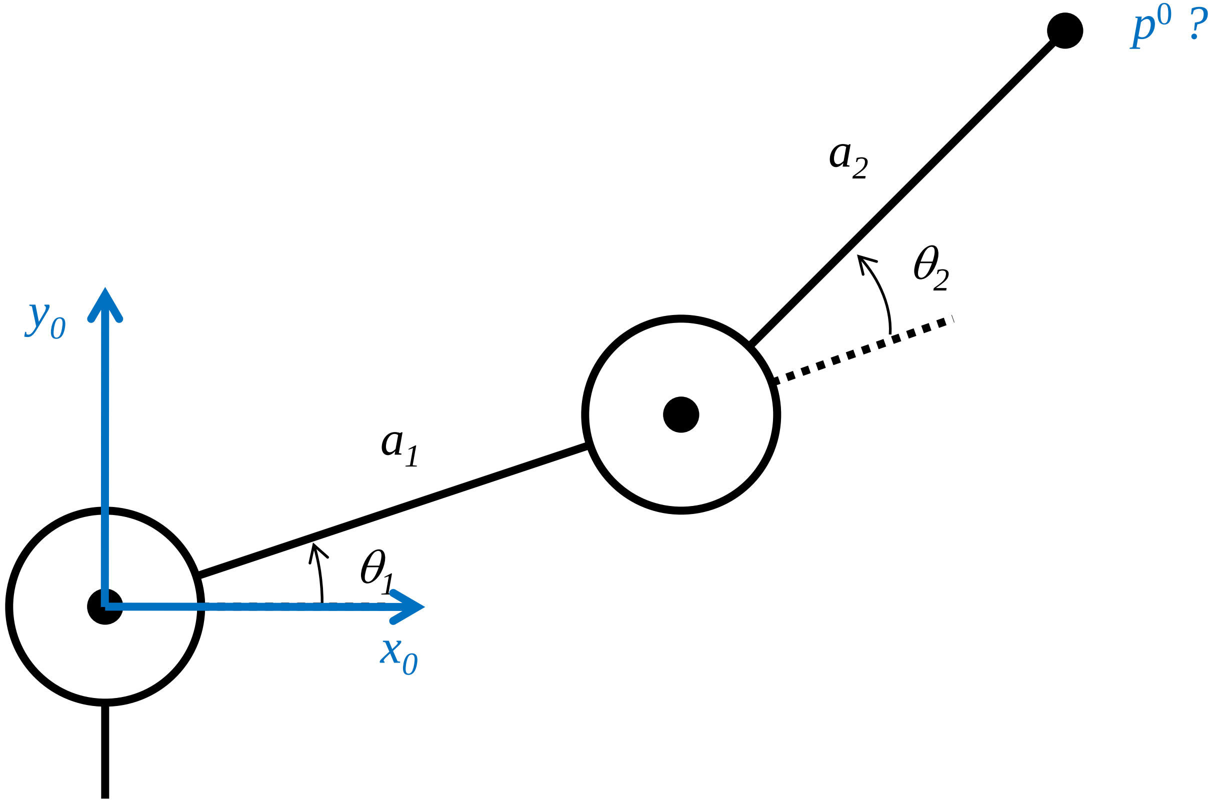


$$q_i = \begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$$

- n joints, $n + 1$ links
- link 0 is fixed (the base)
- joint i connects link $i - 1$ to link i
  - link i moves when joint i is actuated

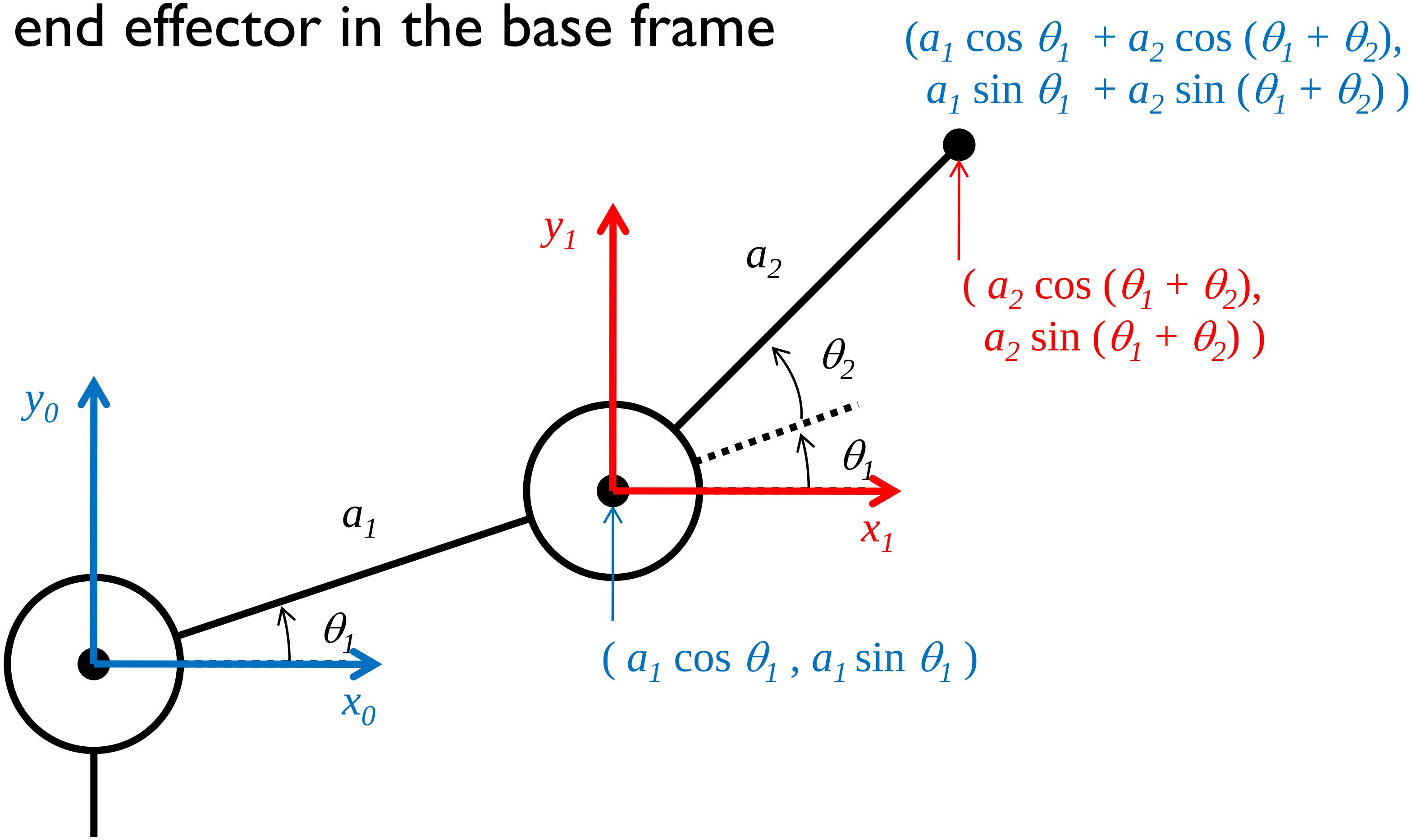
Forward Kinematics

- given the joint variables and dimensions of the links what is the position and orientation of the end effector?



Forward Kinematics

- because the base frame and frame *1* have the same orientation, we can sum the coordinates to find the position of the end effector in the base frame



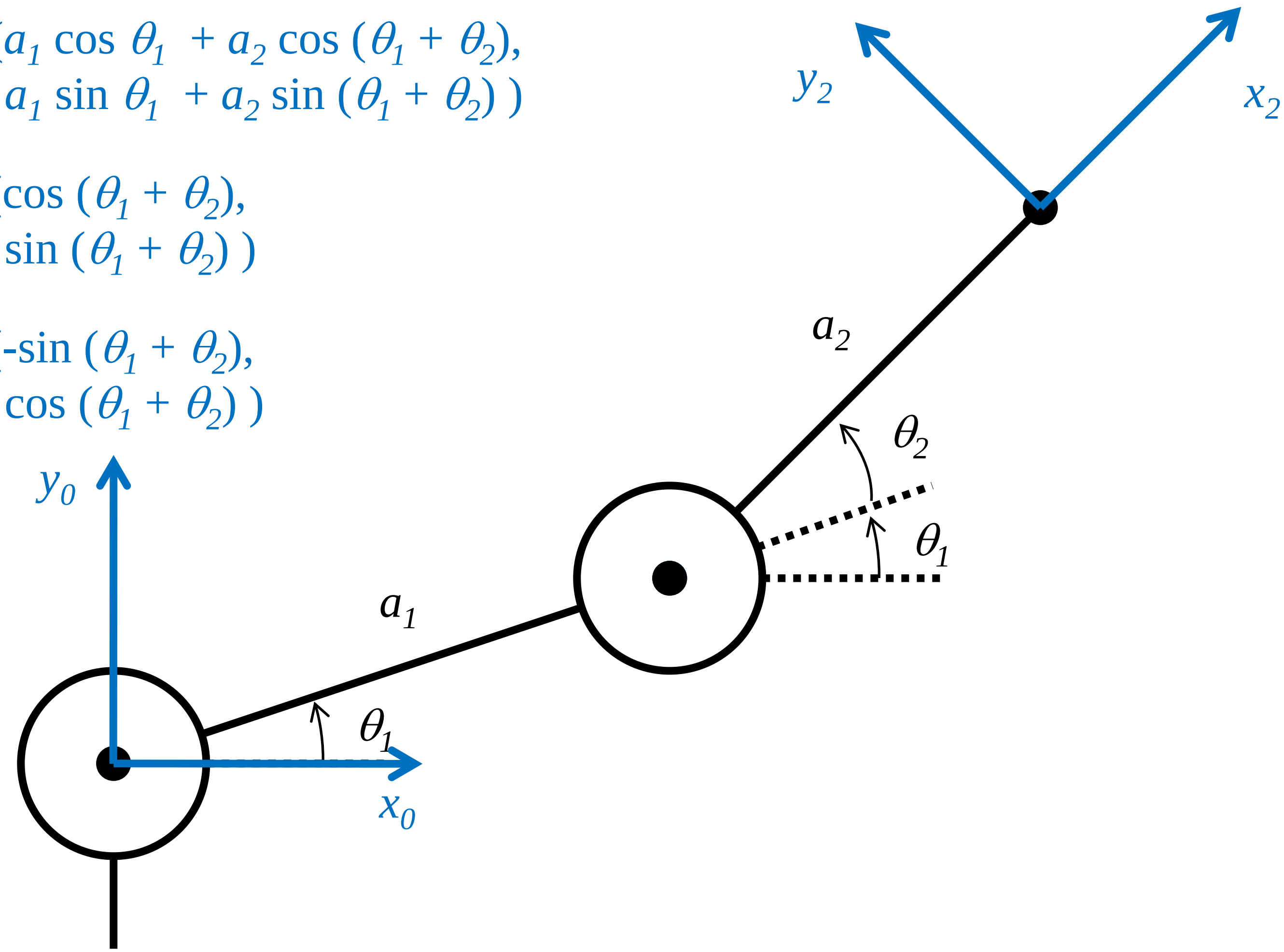
Forward Kinematics

- from Day 02

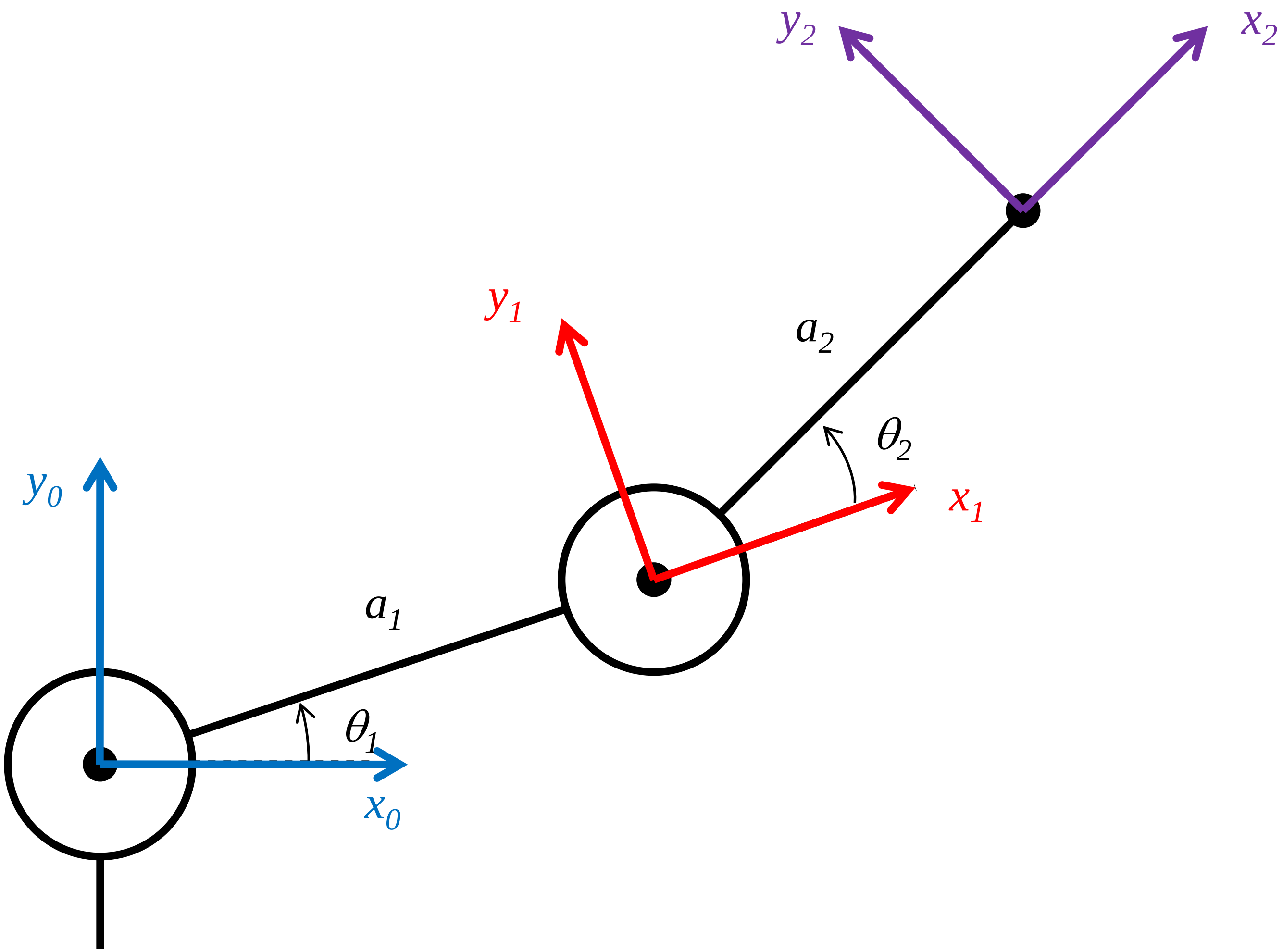
$$p^0 = (a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2),$$
$$a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2))$$

$$x_2 = (\cos(\theta_1 + \theta_2),$$
$$\sin(\theta_1 + \theta_2))$$

$$y_2 = (-\sin(\theta_1 + \theta_2),$$
$$\cos(\theta_1 + \theta_2))$$



Frames



Forward Kinematics

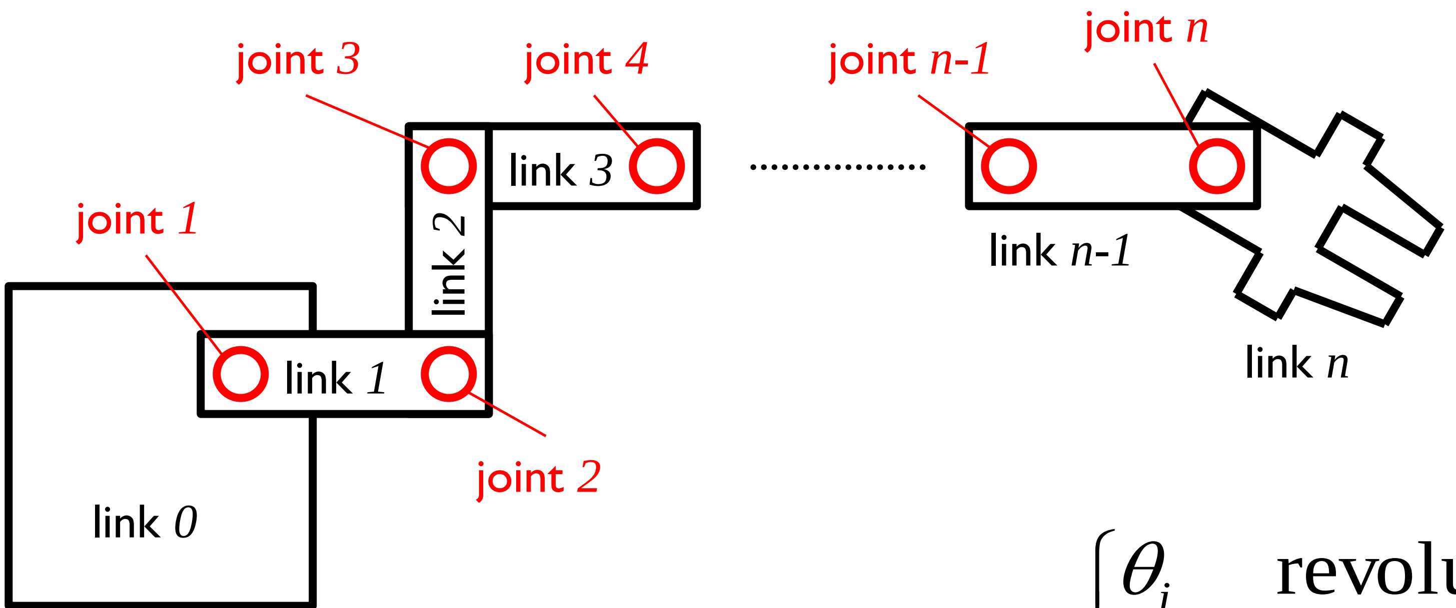
- using transformation matrices

$$T_1^0 = R_{z,\theta_1} D_{x,a_1}$$

$$T_2^1 = R_{z,\theta_2} D_{x,a_2}$$

$$T_2^0 = T_1^0 T_2^1$$

Links and Joints



$$q_i = \begin{cases} \theta_i & \text{revolute} \\ d_i & \text{prismatic} \end{cases}$$

- n joints, $n + 1$ links
- link 0 is fixed (the base)
- joint i connects link $i - 1$ to link i
  - link i moves when joint i is actuated

Forward Kinematics

- attach a frame $\{i\}$ to link i
  - all points on link i are constant when expressed in $\{i\}$
  - if joint i is actuated then frame $\{i\}$ moves relative to frame $\{i - 1\}$
    - motion is described by the rigid transformation

$$T_i^{i-1}$$

    - the state of joint i is a function of its joint variable q_i (i.e., is a function of q_i)

$$T_i^{i-1} = T_i^{i-1}(q_i)$$

- this makes it easy to find the last frame with respect to the base frame

$$T_n^0 = T_1^0 \, T_2^1 \, T_3^2 \cdots T_n^{n-1}$$

Forward Kinematics

- more generally

$$T^{i}_{j} = \begin{cases} T^{i}_{i+1}\, T^{i+1}_{j+2} \ldots T^{j-1}_{j} & \text{if} \quad i < j \\ I & \text{if} \quad i = j \\ \left(T^{i}_{j}\right)^{-1} & \text{if} \quad i > j \end{cases}$$

- the forward kinematics problem has been reduced to matrix multiplication

Forward Kinematics

- Denavit J and Hartenberg RS, "A kinematic notation for lower-pair mechanisms based on matrices." *Trans ASME J. Appl. Mech*, 23:215–221, 1955
  - described a convention for standardizing the attachment of frames on links of a serial linkage
- common convention for attaching reference frames on links of a serial manipulator and computing the transformations between frames

Denavit-Hartenberg

$$T_i^{i-1} = R_{z,\theta_i} T_{z,d_i} T_{x,a_i} R_{x,\alpha_i}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a_i link length

α_i link twist

d_i link offset

θ_i joint angle

Denavit-Hartenberg

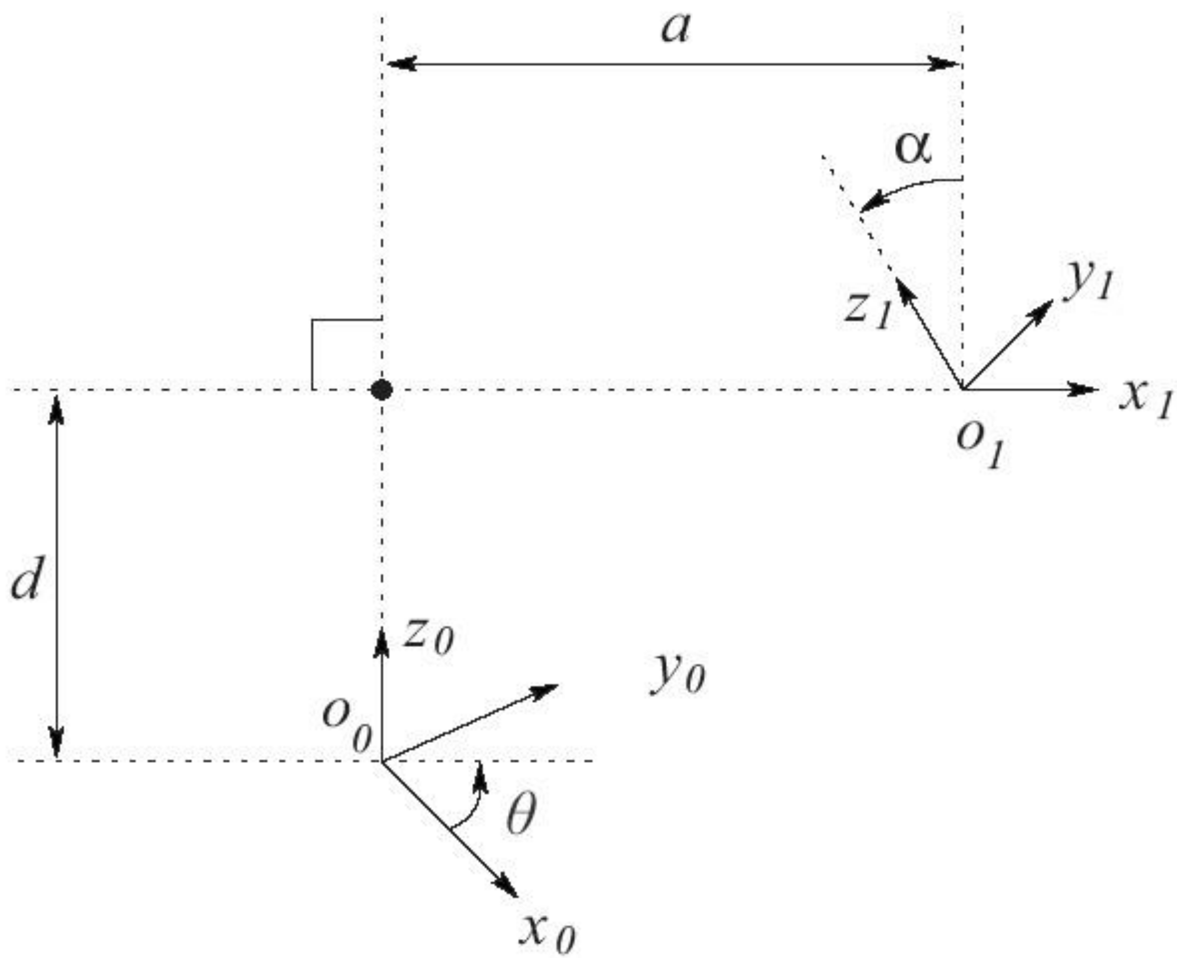


Figure 3.2: Coordinate frames satisfying assumptions DH1 and DH2.

Denavit-Hartenberg

- notice the form of the rotation component

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} \end{bmatrix}$$

  - this does not look like it can represent arbitrary rotations
- can the DH convention actually describe every physically possible link configuration?

Denavit-Hartenberg

- yes, but we must choose the orientation and position of the frames in a certain way

  - (DH1) $\hat{x}_1 \perp \hat{z}_0$

  - (DH2) $\hat{x}_1$ intersects $\hat{z}_0$

- claim: if DH1 and DH2 are true then there exists unique numbers

$$a, d, \theta, \alpha \text{ such that } T_1^{\,0} = R_{z,\theta}\, D_{z,d}\, D_{x,a}\, R_{x,\alpha}$$

Denavit-Hartenberg

- proof: on blackboard in class